Asymptotic freedom and higher derivative gauge theories

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arXiv:2012.15693v2 [hep-th] (click it!)

Affiliation

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Motivation for HD theories

Gauge theories with higher derivatives

- covariant ultraviolet regularizations of gauge theories (Slavnov, Lee, Zinn-Justin)
- very efficient effective theories in strongly correlated regimes of standard QCD
- might provide ultraviolet completions of the Standard Model
- better control over UV perturbative divergences
- super-renormalizable and UV-finite models of gauge theories
- arise as first two leading terms in the continuum limit of Wilson's action of lattice gauge theories

Motivation from QG

HD gauge theories are simpler framework to understand HD QG theories

Motivation from Quantum Gravity

Modest approach:

Let's first quantize matter, put it on curved spacetime background, only later quantize gravitation (Utiyama, De Witt, Shapiro)

Observation:

1-loop off-shell divergences of standard matter theory (with two derivatives) are proportional to R^2 and C^2 on a curved spacetime background. Gravitational counterterms in external background metric needed to be added to the divergent matter effective action are of these types R^2 and C^2 (in $d = 4$), even if the gravitational theory was Einstein-Hilbert Quantum Gravity with R in the action

Conclusion:

These counterterms contain higher derivatives of the background metric. Higher derivatives in QG are inevitable!

HD gauge theories - definition

Action functional

- pure HD YM theory in Euclidean framework in $d = 4$

$$S = \frac{1}{4g^2} \int d^4x \, F^a_{\mu\nu} F^{\mu\nu\, a} + \frac{1}{4g^2 \Lambda^{2n}} \int d^4x \, F^a_{\mu\nu} \, \Delta^n \, F^{\mu\nu\, a} \tag{1}$$

- definition of the Δ operator

$$\Delta = (\Delta)_{\mu\, a}^{\nu\, b} = -D^2 \delta^\nu_\mu \delta^b_a + 2 f^b{}_{ca} \, F_\mu{}^{\nu c} \tag{2}$$

- gauge-covariant derivative

$$D_\mu X^a = \partial_\mu X^a + f^{abc} A^b_\mu X^c \tag{3}$$

- Hodge-covariant Laplacian operator $\Delta = d^*_A d_A + d_A d^*_A$

Useful generalization

$$\Delta \to {}^\lambda\Delta = {}^\lambda\Delta_{\mu\, a}^{\nu\, b} = -\delta^b_a \delta^\nu_\mu D^2 + 2\lambda f^b{}_{ca} \, F_\mu{}^{\nu c} \, . \tag{4}$$

Quantization of HD gauge theories

Covariant formalism

- addition of Lorentz-covariant gauge fixing

$$S_\alpha = \frac{\alpha}{2g^2\Lambda^{2n}} \int d^4x \, \partial^\mu A^a_\mu (-\partial^\sigma \partial_\sigma)^n \partial^\nu A^a_\nu \tag{5}$$

- Faddeev-Popov quantization
- addition of FP ghosts and third ghosts for HD

Power counting of UV-divergences

- superficial degree of divergence

$$\omega \leqslant 4 - 2n(L-1) - E \tag{6}$$

(L - number of loops, $E \geqslant 2$ - number of external gluons,
$2n$ - number of additional derivatives above 2 in standard YM)

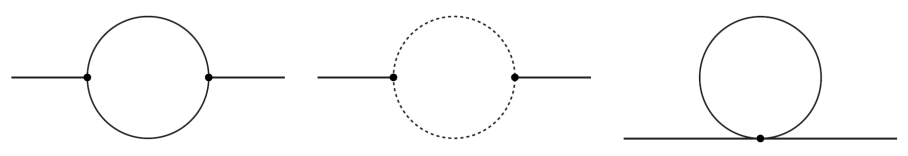
- for $n = 1$ divergences at one- and two-loop level
- for $n > 1$ HD gauge theory is super-renormalizable (divergences only at one-loop)

Computation of UV-divergences

One-loop computation

- in Minkowski spacetime formalism
- using dimensional regularization $\epsilon = 4 - d$
- for 2-pt gluonic function
- from one-loop vacuum polarization diagrams

Feynman diagrams needed for 2-pt function



- middle diagram (with FP ghost) is the same for any $n \geqslant 0$
- last diagram vanishes in DIMREG for $n = 0$

Results for 2-pt function

UV-divergent part

- vacuum polarization function

$$\Gamma^{ab}_{\mu\nu}(p) = -c_n \frac{C_2(G)}{16\pi^2\epsilon} i\delta^{ab}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right) \tag{7}$$

- coefficients c_n

$$c_0 = \alpha - \frac{13}{3}, \quad c_1 = -\frac{43}{3} \text{ and } c_n = \frac{29}{3} - 23n + 5n^2 \text{ for } n \geqslant 2 \tag{8}$$

- $n = 1$ results of Babelon-Namazie
- $n \geqslant 2$ results of Asorey-Falceto
- perturbative UV-form of modified beta functions for $n \geqslant 1$

$$\beta_n = c_n \frac{g^3 C_2(G)}{32\pi^2} \tag{9}$$

Renormalization

Renormalized action

- 2-derivative action with counterterms

$$S_{\rm count} = c_n \frac{C_2(G)}{128\pi^2}\left(\frac{2}{\epsilon} + \log\frac{\Lambda^2_{\rm QCD}}{\Lambda^2}\right) F^a_{\mu\nu}F^{\mu\nu\, a} \tag{10}$$

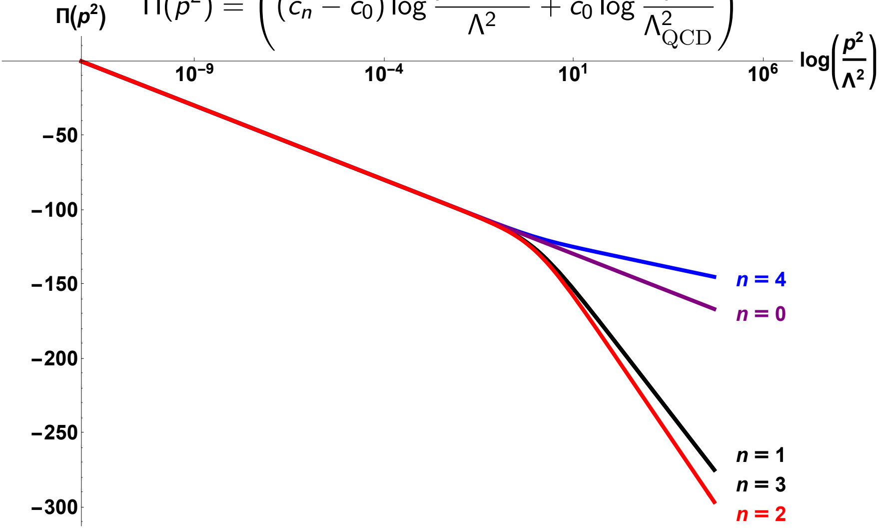
- added a finite counterterm to the minimal renormalization $\log\frac{\Lambda^2_{\rm QCD}}{\Lambda^2}$
- recover in the IR the renormalized two-point function of QCD
- no gluon wave-function renormalization for $n \geqslant 1$
- renormalization to every loop order in $n = 0$
- only one-loop renormalization for $n > 1$
- for $n \geqslant 1$ no α-dependence

Renormalized 2-pt function

$$\Gamma^{ab}_{\mu\nu}(p) = -\frac{C_2(G)}{32\pi^2} i\delta^{ab}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\Pi(p^2), \tag{11}$$

Polarization function $\Pi(p^2)$ vs. logarithmic energy scale

$$\Pi(p^2) = \left((c_n - c_0) \log \frac{p^2 + \Lambda^2}{\Lambda^2} + c_0 \log \frac{p^2}{\Lambda^2_{\rm QCD}} \right)$$



RG flow in HD gauge theories

Beta functions

$$\beta_{\rm UV} = c_n \frac{g^3 C_2(G)}{32\pi^2} \quad \text{and} \quad \beta_{\rm IR} = -\frac{22}{3}\frac{g^3 C_2(G)}{32\pi^2} \tag{12}$$

Asymptotic freedom $\beta < 0$ in UV

coefficients in theories $n \leqslant 4$

$$\widetilde{c}_0 = -\frac{22}{3}, \quad c_1 = -\frac{43}{3}, \quad c_2 = -\frac{49}{3}, \quad c_3 = -\frac{43}{3}, \quad c_4 = -\frac{7}{3} \tag{13}$$

RG running

flow of the bare coupling constant

$$g^2_{\rm bare}(\mu) = g^2 \left(1 - \frac{g^2 C_2(G)}{(4\pi)^2} c_n \log \mu/\Lambda\right)^{-1} \tag{14}$$

RG flow in HD gauge theories

RG flow of the HD scale Λ

- no-renormalization of the HD term

$$\frac{1}{4g^2\Lambda^{2n}}\int d^4x\, F^a_{\mu\nu}\,\Delta^n\,F^{\mu\nu\,a} \tag{15}$$

- constancy of the front coefficient

$$g^2\Lambda^{2n} = g^2_{\rm bare}\Lambda^{2n}_{\rm bare} = \text{const} \quad \text{hence} \quad \beta_\Lambda = -\frac{\Lambda}{ng}\beta_n \tag{16}$$

UV regime

- if the g coupling is asymptotically free (AF)

$$g_{\rm bare} \to 0 \tag{17}$$

- then the running $\Lambda_{\rm bare}$ scale parameter must run

$$\Lambda_{\rm bare} \to +\infty, \quad m_{\rm gh} \sim \Lambda_{\rm bare} \to +\infty \tag{18}$$

Generalized results

Generalized ${}^{\lambda}\Delta$ operator

$$\Delta \to {}^{\lambda}\Delta = {}^{\lambda}\Delta_{\mu\ a}^{\nu\ b} = -\delta_a^b \delta_\mu^\nu D^2 + 2\lambda f^b{}_{ca} F_\mu{}^{\nu c} \quad (19)$$

Beta functions

- general expression

$$\beta_{\rm UV} = c_n \frac{g^3 C_2(G)}{32\pi^2} \quad (20)$$

- for $n = 1$

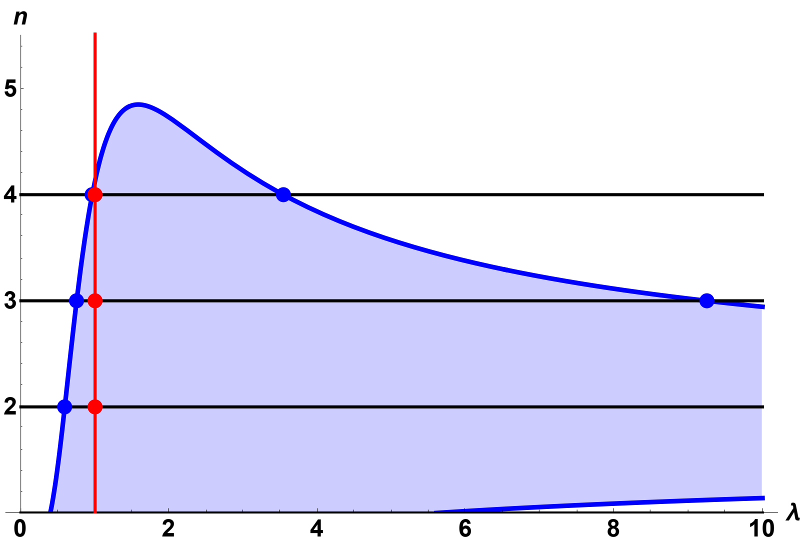
$$c_1 = \frac{38}{3} - 18\lambda - 9\lambda^2 \quad (21)$$

- for $n \geqslant 2$

$$c_n = -\frac{7}{3} + 5n + 4n^2 - \left(4 + 10n + 4n^2\right)\lambda + \left(16 - 18n + 5n^2\right)\lambda^2 \quad (22)$$

For some values λ_* of the λ parameter possibility of UV-finite theory

Region of AF HD gauge theories



UV-finite gauge theories

Values of λ_* with ${}^{\lambda}\Delta$ operator

$$\begin{array}{llll} n=1 & \lambda_1 = & -2.55 & \lambda_2 = 0.55 \\ n=2 & \lambda \;\, = & 0.59 & \\ n=3 & \lambda_1 = & 0.75 & \lambda_2 = 9.25 \\ n=4 & \lambda_1 = & 0.96 & \lambda_2 = 3.54 \end{array}$$

contains cubic ($O(F^3)$) and quartic killers ($O(F^4)$)

Quartic killers

Addition of pure gauge-covariant quartic killers Modesto, Piva

$$S_{\mathrm{UV,\,fin}} = \int d^4x \frac{1}{4g^2} \mathrm{tr} \left[\mathbf{F} \left(\frac{D^2}{\Lambda^2} \right)^n \mathbf{F} + \frac{s_g}{\Lambda^4} (\mathbf{F}^2) \left(\frac{D^2}{\Lambda^2} \right)^{n-2} (\mathbf{F}^2) \right] \tag{23}$$

with value of the killer coefficient

$$s_g = 2\pi^2 \frac{\beta_n}{g^3} = \frac{C_2(G)}{16} \left(-\frac{7}{3} + 5n + 4n^2 \right) \tag{24}$$

Conclusions

Final comments

- super-renormalizable HD gauge theory for $n \geqslant 2$
- UV asymptotically free theory for $\lambda = 1$ and $n < 5$
- decoupling of ghosts in AF theory ($m_{\rm gh} \sim \Lambda_{\rm bare} \to +\infty$)
- consistent UV-completion of HD gauge theory puts constraints on n and λ
- possibility for UV-finite theories

Applications

- expansion of the continuum limit of lattice gauge theory, HD QCD
- axion physics phenomenology
- more room for GUT unification
- Lorentz- and gauge-covariant regularization of YM theories
- lessons for Quantum Gravity

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Thank you!

Obrigado!

Arigato!

ありがとうございました