Regular solutions in weak-field infinite-derivative theories: Green function approach

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Based on work with Valeri P. Frolov, Andrei Zelnikov, Jose Pinedo Soto, Ivan Kolar

Quantum Gravity, Higher Derivatives & Nonlocality
Online workshop, Tokyo Institute of Technology, Japan
Thursday, March 11, 2021, 12:30pm

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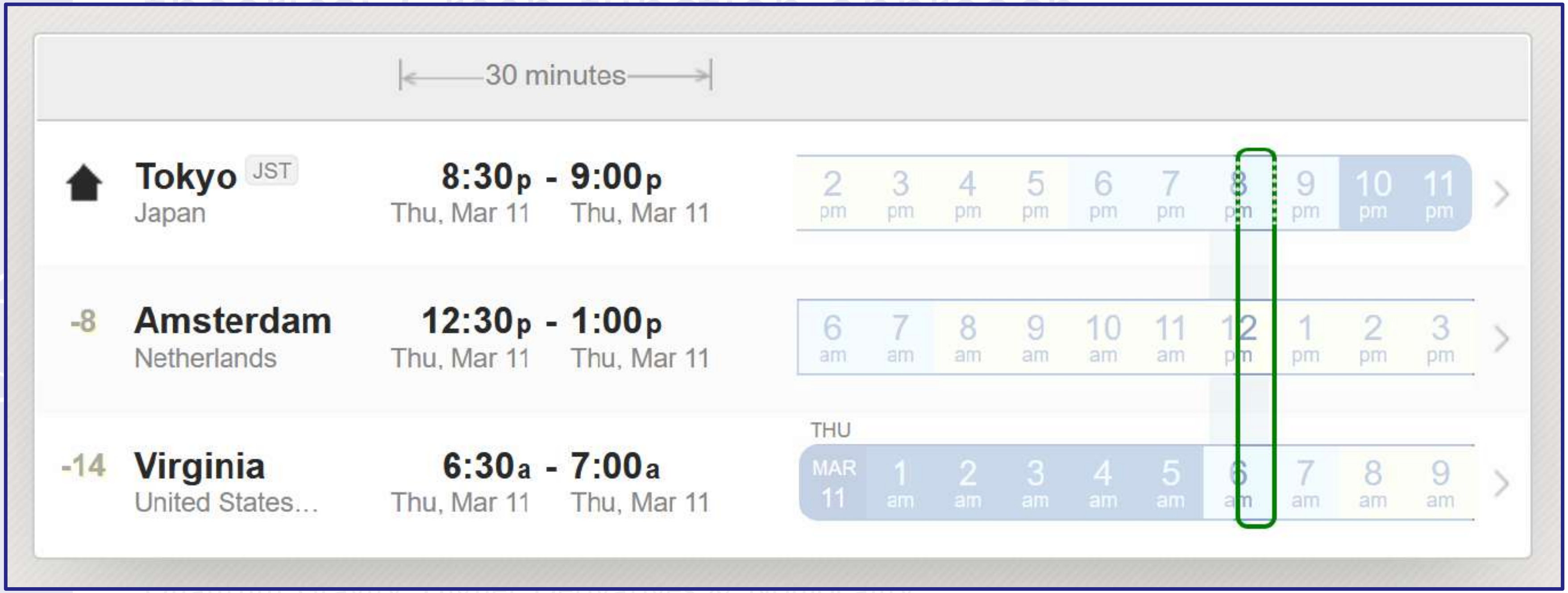

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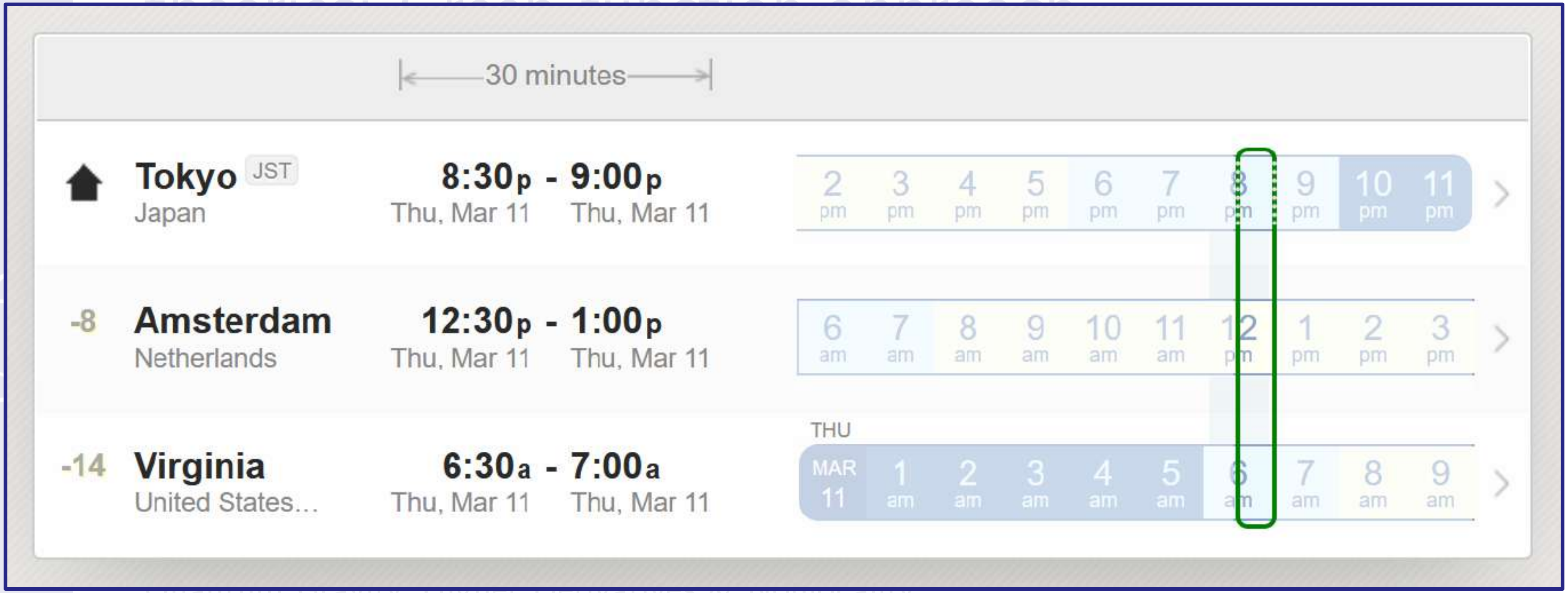

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JB 2009.10856 [gr-qc]

Motivation: let's develop a classical "non-local intuition"

Use Green function approach to study wide field of non-local weak-field problems:

- static, stationary, and ultrarelativistic particles and branes in gravity;
- Maxwell theory; quantum scattering and particle production on delta potential;
- vacuum fluctuations around delta potential in QFT;
- fluctuation-dissipation theorem at finite temperature;
- corrections to black hole entropy in 2D via Polyakov effective action.

Motivation: let's develop a classical "non-local intuition" (and address some of Tuesday's & Wednesday's comments)

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- Maxwell theory; quantum scattering and particle production on delta potential;
- vacuum fluctuations around delta potential in QFT;
- fluctuation-dissipation theorem at finite temperature;
- corrections to black hole entropy in 2D via Polyakov effective action.

Richard Woodard: Doesn't an integral kernel imply that we know the field infinitely far away?

Terry Tomboulis: Cauchy problem is ill-defined in non-local theories.

Frank Saueressig: infinite-derivative $\frac{1}{k^2[1+k^2\tanh(k^2)]}$-toy model

Irina Aref'eva: heat kernel methods

Philip Mannheim: How do we guarantee the correct IR behavior?

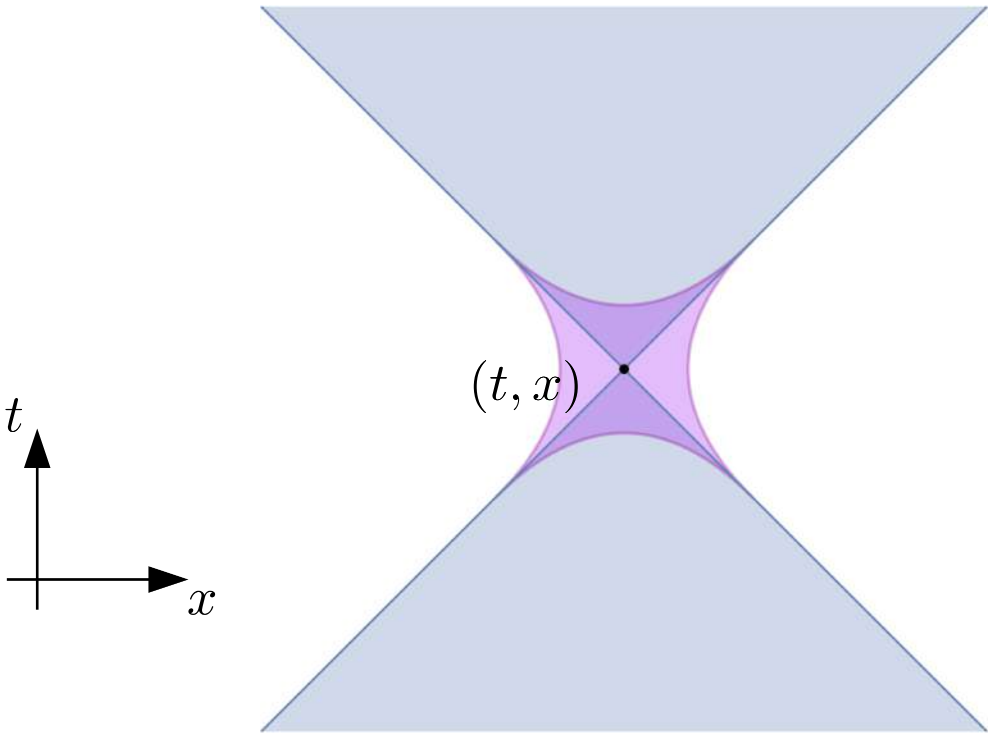
Robert Brandenberger: "Smearing out" can be a regularization mechanism.

PART I

NON-LOCAL GREEN FUNCTIONS IN (3+1)D

model ▪ asymptotic causality ▪ uniformly accelerated particle

Non-locality at small scales? Not really.



$$-(t' - t)^2 + (x' - x)^2 \leq \ell^2$$

It is difficult to define a Lorentz-invariant notion of a "small region" in spacetime.

Simple infinite-derivative non-local model (1/3)

Simple model: classical linear scalar infinite-derivative theory on flat spacetime $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2$.

$$f(\ell^2\Box)\Box\varphi(t,\boldsymbol{x}) = \sigma(t,\boldsymbol{x})$$

form factor — scale of non-locality — external source

Notation: $\Box \equiv -\dfrac{\partial^2}{\partial t^2} + \triangle\,, \quad \triangle \equiv \displaystyle\sum_{i=1}^{d} \frac{\partial^2}{\partial {x^i}^2} \overset{S^{d-1}}{=} \frac{1}{r^{d-1}}\frac{\partial}{\partial r}\left(r^{d-1}\frac{\partial}{\partial r}\right)$

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Important requirements on the form factor:

- $f(z) \neq 0$: Can be inverted everywhere, no new zero modes (classical version of "ghost-free").
  Important difference to higher-derivative models.

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- $f(z) \neq 0$: Can be inverted everywhere, no new zero modes (classical version of "ghost-free").
  Important difference to higher-derivative models.

- $f(0) = 1$: Guarantees the correct far-distance asymptotics ("IR properties")
  and the correct local limit $\ell \to 0$.
  Also removes effects on on-shell quantities, $f(\ell^2\Box)X = f(0)X = X$.

Simple infinite-derivative non-local model (3/3)

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How to find a solution of this equation?

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$$f(\ell^2\Box)\Box\varphi(t,\boldsymbol{x}) = \sigma(t,\boldsymbol{x})$$

How to find a solution of this equation?

$$f(\ell^2\Box)\Box\mathcal{G}(t'-t,\boldsymbol{x}'-\boldsymbol{x}) = -\delta(t'-t)\delta^{(3)}(\boldsymbol{x}'-\boldsymbol{x})$$

$$\varphi(t,\boldsymbol{x}) = -\int\limits_{-\infty}^{\infty}\mathrm{d}t\int\limits_{\mathbb{R}^d}\mathrm{d}^d x\,\mathcal{G}(t-t',\boldsymbol{x}-\boldsymbol{x}')\sigma(t',\boldsymbol{x}')$$

I will not discuss uniqueness of solutions, or initial value problem.

Instead, I will focus on the causal properties of the non-local infinite-derivative Green function.

Causal properties (3+1)D non-local Green function

$$\mathcal{G}(t'-t,\boldsymbol{x}'-\boldsymbol{x}) = \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega(t'-t)} \int\limits_{\mathbb{R}^3} \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x})} I(\omega,\boldsymbol{k})$$

$$I(\omega,\boldsymbol{k}) = -\frac{1}{\omega^2-\boldsymbol{k}^2}\frac{1}{f[\ell^2(\omega^2-\boldsymbol{k}^2)]} = \frac{1}{\omega^2-\boldsymbol{k}^2}\left(1-\frac{1}{f[\ell^2(\omega^2-\boldsymbol{k}^2)]}\right) - \frac{1}{(\omega-i\epsilon)^2-\boldsymbol{k}^2}$$

Sum of two pieces.

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Sum of two pieces. **Non-local modification** has no poles.

Local part needs to be regularized, which gives rise to **causal properties**.

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$$\mathcal{G} = G + \Delta\mathcal{G}$$

$$G^{\mathrm{R}}(t'-t,\boldsymbol{x}'-\boldsymbol{x}) = \frac{1}{2\pi}\delta^{(2)}(s^2)\,\theta(t'-t)\,, \quad s^2 \equiv -(t'-t)^2 + (\boldsymbol{x}'-\boldsymbol{x})^2$$

Retarded local Green function non-trivial only on future light cone.

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Retarded local Green function non-trivial only on future light cone. What about $\Delta\mathcal{G}$?

Properties of a non-local modification (1/3)

Consider a concrete model where Green function exists, $f(\ell^2\Box) = \exp\left[(-\ell^2\Box)^2\right]$.

$$\Delta\mathcal{G}(|s^2|) = \frac{|s^2|}{1024\pi^2\ell^4} G^{20}_{03}\left({}_{-\frac{1}{2}, -\frac{1}{2};\ -1} \middle| \frac{s^4}{256\ell^4} \right)$$

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Asymptotics for large and small arguments:

$$\Delta\mathcal{G}(|s^2| \ll 1) = \frac{1}{32\pi^{5/2}\ell^2}\left[2 - 3\gamma - \log\left(\frac{s^4}{64\ell^4}\right)\right]$$

$$\Delta\mathcal{G}(|s^2| \gg 1) = \frac{1}{2\sqrt{3}\pi^2|s^2|} \sin\left(\frac{3\sqrt{3}s^{4/3}}{8\cdot 2^{2/3}\ell^{4/3}}\right) \exp\left(-\frac{3s^{4/3}}{8\cdot 2^{2/3}\ell^{4/3}}\right)$$

Buoninfante *et al.* 1805.03559 [hep-th], JB 2009.10856 [gr-qc], Kolar & JB 2102.07843 [hep-th]

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Two important observations:

- Exponential suppression if $|s^2| \to \infty$. Example: $t' - t \to -\infty$ at constant $\boldsymbol{x}' - \boldsymbol{x}$.
  This is Bryce DeWitt's **asymptotic causality** criterion on Green functions.
- Logarithmic **divergence** in coincidence limit (or for null separation).

Uniformly accelerated source

Find the retarded field of uniformly accelerated source in (3+1)D Minkowski spacetime:

$$f(\ell^2\Box)\Box\varphi(t,\boldsymbol{x}) = \sigma(t,\boldsymbol{x})$$

$$f(\ell^2\Box) = \exp\left[(-\ell^2\Box)^2\right] \,,$$

$$\sigma(t,\boldsymbol{x}) = 2\mu\alpha\,\delta(-t^2+z^2-\alpha^2)\delta(x)\delta(y)\theta(z+t)\theta(z-t)\,.$$

Uniformly accelerated source (2/3)

Find the retarded field of uniformly accelerated source in (3+1)D Minkowski spacetime:

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Kolar & JB 2102.07843 [hep-th]

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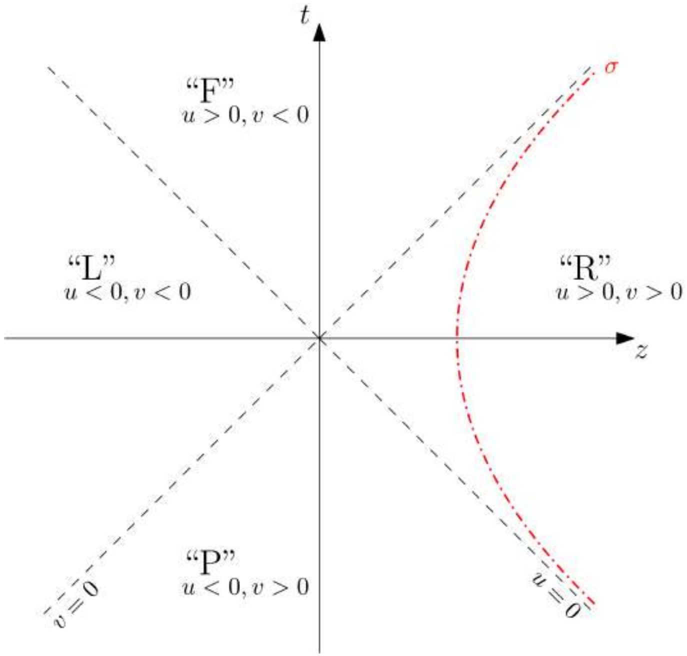

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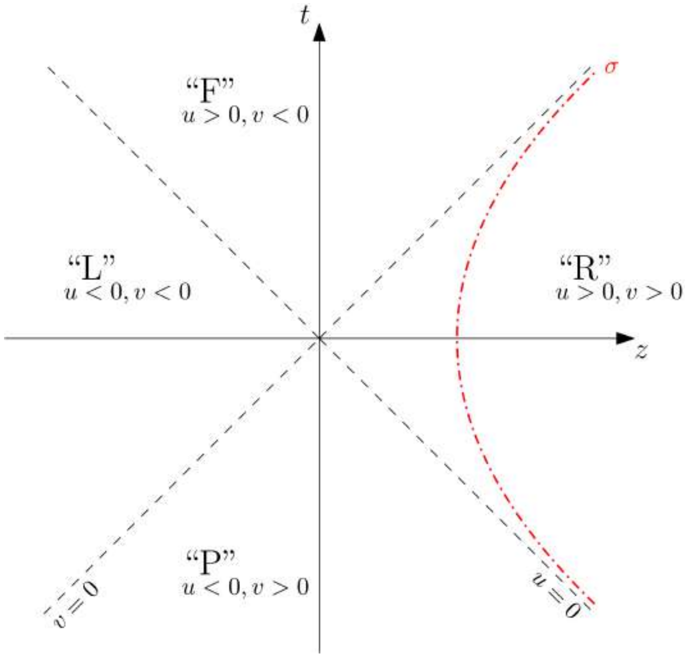

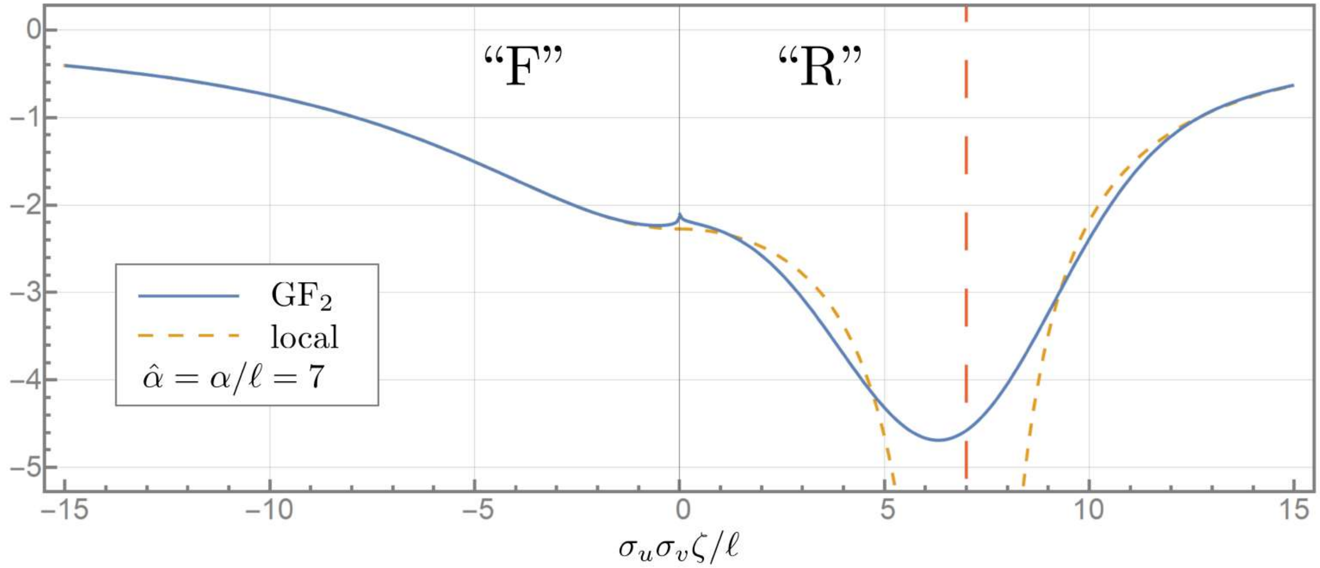

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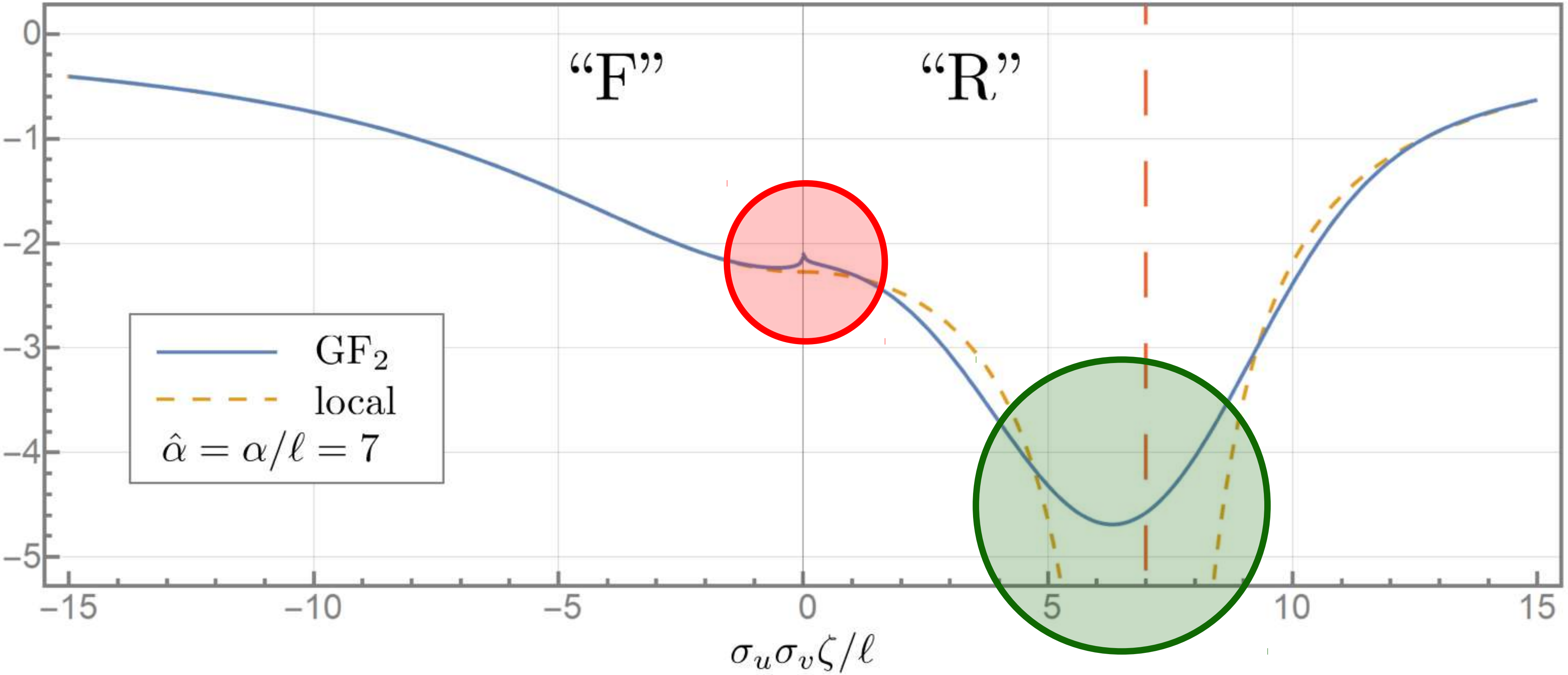

PART II

NON-LOCAL GREEN FUNCTIONS IN 3D

integral kernels ▪ weak-field potentials ▪ heat kernel representation

Simple(r) infinite-derivative non-local model

Simple model: classical linear scalar *static* infinite-derivative theory on flat space $\mathrm{d}s^2 = \mathrm{d}\boldsymbol{x}^2$.

$$f(\ell^2 \triangle) \triangle \varphi(\boldsymbol{x}) = \sigma(\boldsymbol{x})$$

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$$f(\ell^2 \triangle)\, \triangle\, \mathcal{G}(\boldsymbol{x}' - \boldsymbol{x}) = -\delta^{(3)}(\boldsymbol{x}' - \boldsymbol{x})$$

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Let us study the properties of this non-local spatial Green function.

Regularity of the non-local spatial Green function

$$\mathcal{G}(\boldsymbol{x}' - \boldsymbol{x}) = \frac{1}{2\pi^2 |\boldsymbol{x}' - \boldsymbol{x}|} \int_0^\infty \frac{\mathrm{d}k}{k} \frac{\sin(k|\boldsymbol{x}' - \boldsymbol{x}|)}{f(-\ell^2 k^2)}$$

Coincidence limit:

$$\mathcal{G}(\boldsymbol{x}' \to \boldsymbol{x}) = \frac{1}{2\pi^2} \int_0^\infty \mathrm{d}k \, \frac{1}{f(-\ell^2 k^2)} \; - \; c_2[f] \, |\boldsymbol{x}' - \boldsymbol{x}|^2 \, , \quad c_2 > 0$$

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Green function regularity dictated by large-momentum properties of form factor $f(-\ell^2 k^2)$.
Fields of δ-concentrated sources are proportional to $\mathcal{G}(r)$, and therefore **regular** at $r \equiv |\boldsymbol{x}| \to 0$.

This also applies to many higher-derivative theories, as long as the integral converges.

Asymptotics of the non-local spatial Green function

$$\mathcal{G}(\boldsymbol{x}' - \boldsymbol{x}) = \frac{1}{2\pi^2|\boldsymbol{x}' - \boldsymbol{x}|} \int_0^\infty \frac{\mathrm{d}k}{k} \sin(k) \frac{1}{f\left(-\frac{\ell^4 k^2}{|\boldsymbol{x}'-\boldsymbol{x}|^2}\right)}$$

Asymptotic limit (more accurately, $|\boldsymbol{x}' - \boldsymbol{x}|/\ell \to \infty$):

$$\mathcal{G}(|\boldsymbol{x}' - \boldsymbol{x}| \to \infty) = \frac{1}{2\pi^2|\boldsymbol{x}' - \boldsymbol{x}|} \int_0^\infty \mathrm{d}k \frac{\sin k}{k} \frac{1}{f(0)} = \frac{1}{4\pi|\boldsymbol{x}' - \boldsymbol{x}|}$$

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The condition $f(0) = 1$ guarantees the correct far-distance limit.

Fields of δ-concentrated sources are proportional to $\mathcal{G}(r)$, and reproduce local theory as $r \to \infty$.

This also applies to many higher-derivative theories.

Heat kernel representation of static Green function

Non-local spatial Green function in d dimensions can be expressed as double Fourier transform:

$$\mathcal{G}_d(r) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\eta}{\eta} \frac{1}{f(-\ell^2\eta)} \int\limits_{-\infty}^{\infty} \mathrm{d}\tau \mathcal{K}_d(r|\tau) e^{i\eta\tau} \,, \qquad \mathcal{K}_d(r|\tau) = \frac{1}{(4\pi i\tau)^{d/2}} e^{i\frac{r^2}{4\tau}}$$

$$\triangle \mathcal{K}_d(r|\tau) = -i\partial_\tau \mathcal{K}_d(r|\tau)$$

JB, Frolov, Pinedo Soto 2004.07420 [gr-qc]

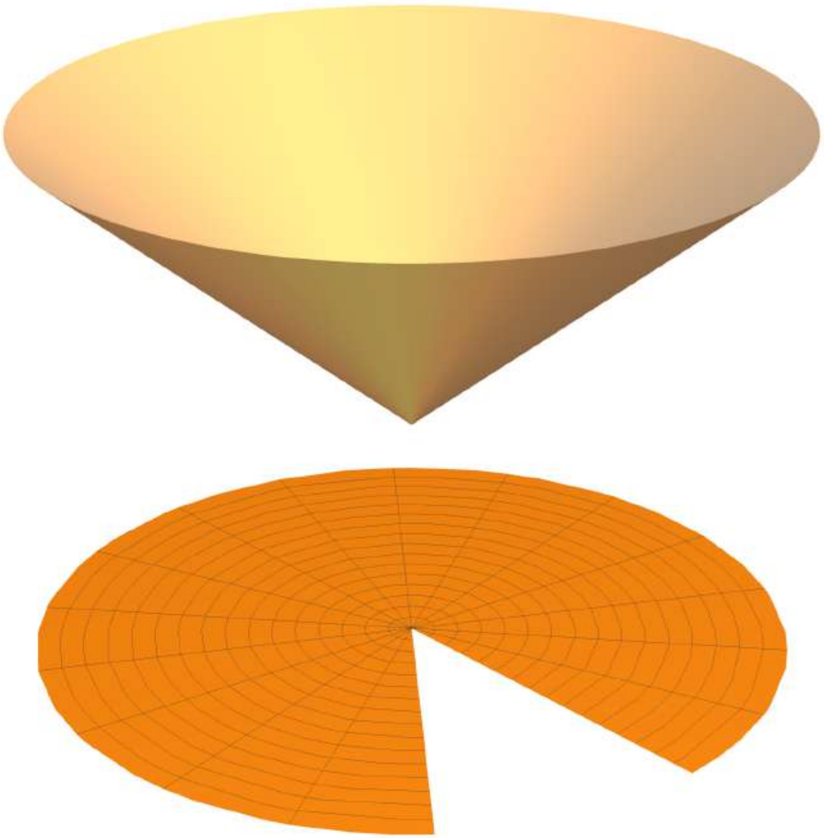
Regularity properties are difficult to extract from this representation.

Works for wide class of (if not all) non-local form factors, not just for $\exp(-\ell^2\triangle)$.

Useful for constructing regular gyraton solutions in Penrose limit.

Not clear if it works in spacetime, heat equation for hyperbolic operator has different properties.

Field of a cosmic string in weak-field local gravity

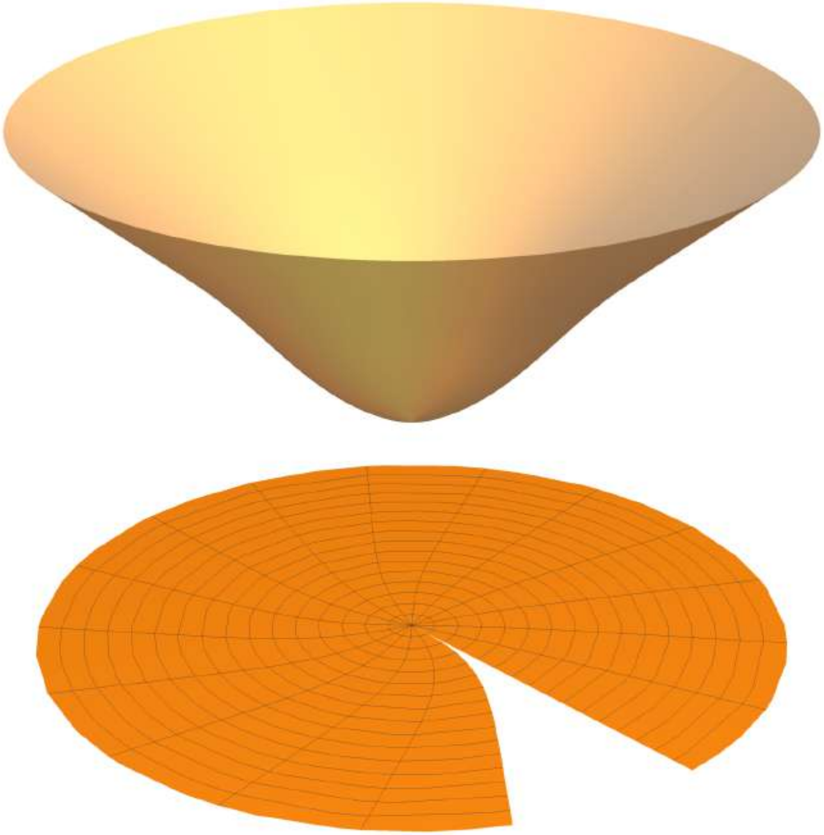


Properties:

- locally flat
- curvature singularity along z-axis
- angle deficit around z-axis

JB 2003.13847 [gr-qc]
Honorable Mention GRF Essay Competition 2020

Field of a cosmic string in weak-field non-local gravity



Properties in local theory:

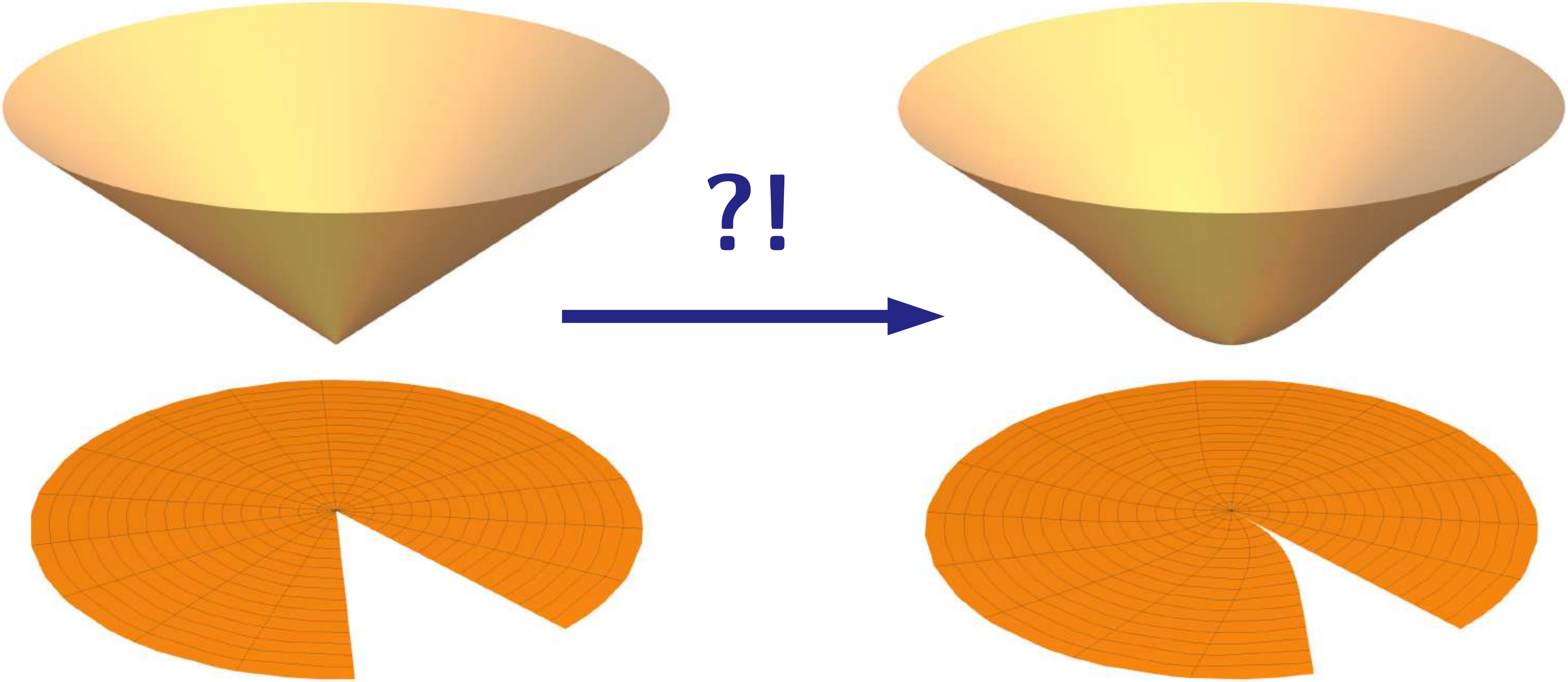
- locally flat
- curvature singularity along z-axis
- angle deficit around z-axis

Properties in **non-local** theory:

- asymptotically flat
- curvature smeared along z-axis
- r-dependent deficit around z-axis

JB 2003.13847 [gr-qc]
Honorable Mention GRF Essay Competition 2020

Field of a cosmic string in non-local gravity



Smearing functions and integral kernels

$$K(\boldsymbol{x}-\boldsymbol{y}) = \frac{1}{f(\ell^2\triangle)}\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) = \frac{1}{2\pi^2|\boldsymbol{x}-\boldsymbol{y}|}\int\limits_0^\infty k\,\mathrm{d}k\,\frac{\sin(k|\boldsymbol{x}-\boldsymbol{y}|)}{f(-\ell^2k^2)}$$

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Integral kernel can be interpreted as the effective energy density of a point particle:

$$f(\ell^2\triangle)\,\triangle\,\varphi(\boldsymbol{x}) = 4\pi m\,\delta^{(3)}(\boldsymbol{x}) \quad \Rightarrow \quad \triangle\varphi(\boldsymbol{x}) = \frac{4\pi m}{f(\ell^2\triangle)}\delta^{(3)}(\boldsymbol{x}) = 4\pi m K(\boldsymbol{x})$$

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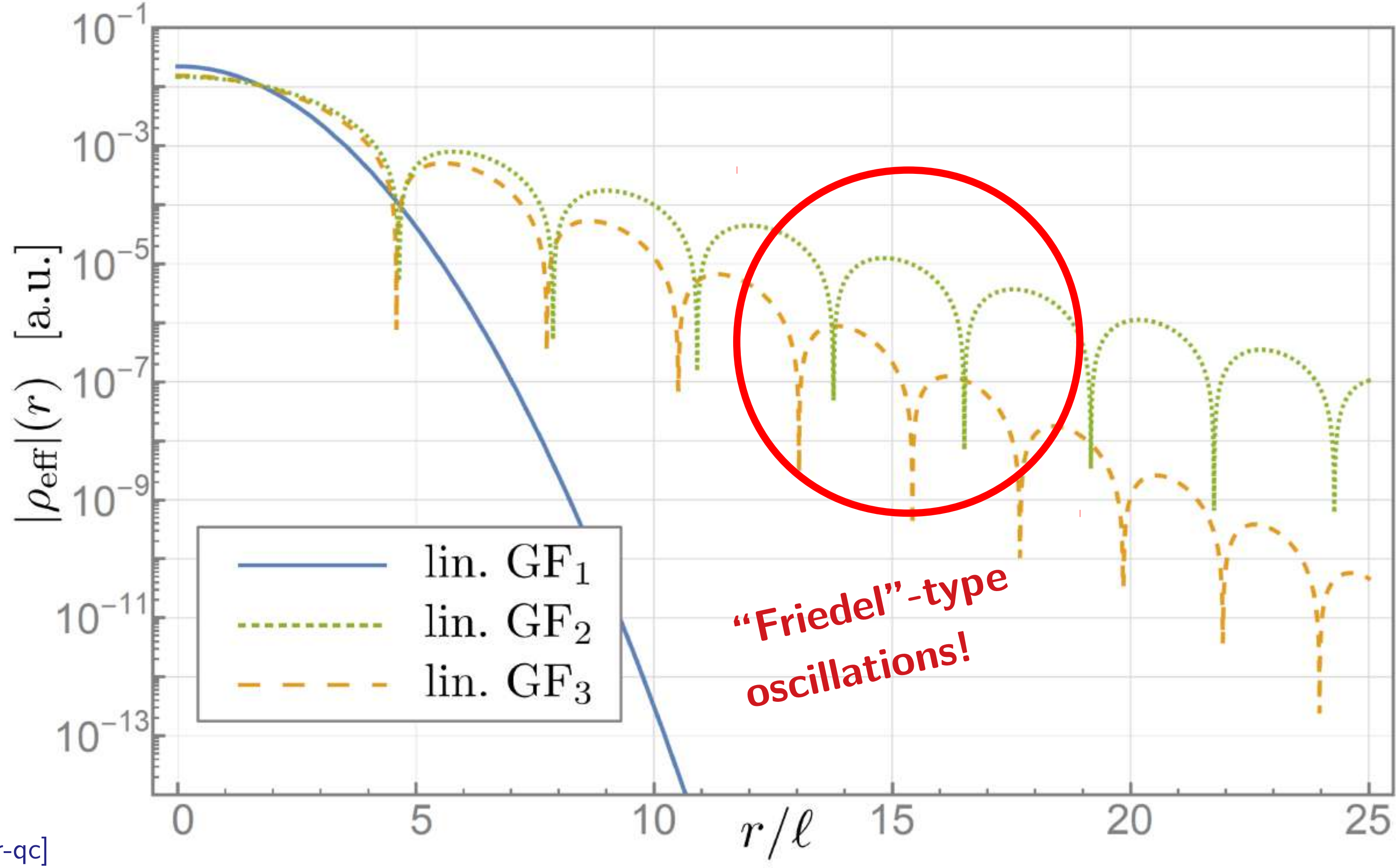
Has to be a nascent/"smeared" δ-distribution.

Can verify explicitly in a simple case of $f(\ell^2\triangle) = \exp(-\ell^2\triangle)$:

$$K(\boldsymbol{x}'-\boldsymbol{x}) = \frac{1}{(4\pi\ell^2)^{d/2}}e^{-\frac{|\boldsymbol{x}'-\boldsymbol{x}|^2}{4\ell^2}} \;\rightarrow\; \delta^{(d)}(\boldsymbol{x}'-\boldsymbol{x}) \qquad \text{in the limit } \;\ell\rightarrow 0\,.$$

Some surprises in integral kernels



JB 1804.00225 [gr-qc]
Honorable Mention GRF Essay Competition 2018

This happens in other models, too.

Approximated static case of Frank Saueressig's toy model from Tuesday:

$$\frac{1}{-k^2 f(-\ell^2 k^2)} = -\frac{1}{k^2[1+\ell^2 k^2 \tanh(\ell^2 k^2)]} \sim -\frac{1}{k^2\left(1+\ell^2 k^2 \frac{\ell^2 k^2}{1+\ell^2 k^2}\right)}$$

Read off the form factor $f(-\ell^2 k^2) = 1 + \dfrac{\ell^4 k^4}{1+\ell^2 k^2}$, and compute static Green function:

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$$\mathcal{G}(r) = \frac{3 + e^{-\sqrt{3}r/(2\ell)}\left[-3\cos\left(\frac{r}{2\ell}\right) + \sqrt{3}\sin\left(\frac{r}{2\ell}\right)\right]}{12\pi r}$$

Oscillatory as well. Can be interpreted in higher-derivative theories, but in non-local case?

Conclusions

What I have shown:

- Can construct a wide class of regular solutions to non-local linear problems.
- (Hopefully) generate some non-local intuition.

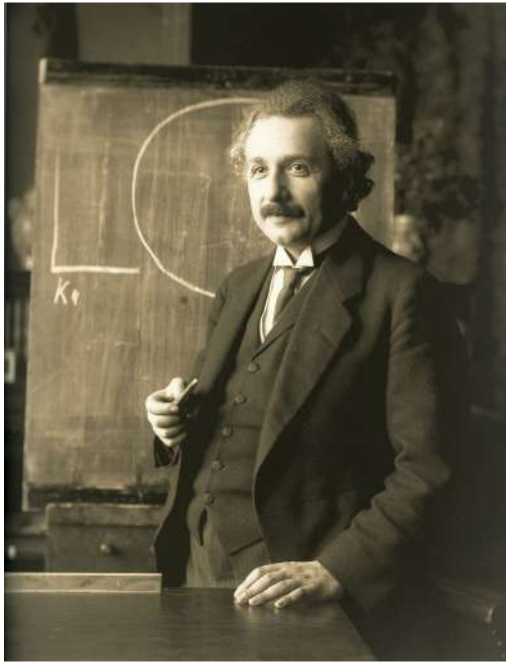
What I have not shown:

- Can apply similar methods in partial Fourier transformed situations.
- Particle creation and scattering in quantum mechanics, use asymptotic locality to define in- and out-states (JB, Frolov, Zelnikov 1805.0187 [hep-th] & 2011.12929 [hep-th]).

What could be next?

- Exact solutions in classical non-linear non-local gravity.
- Perturbation theory via Lippmann–Schwinger equation.
- ...

An afterthought



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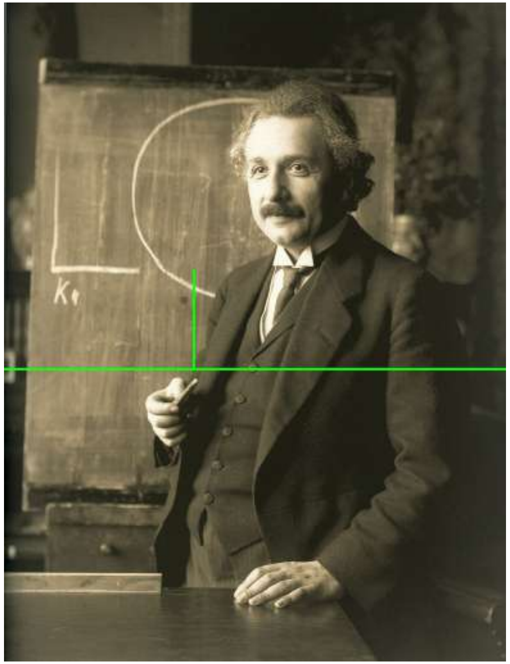

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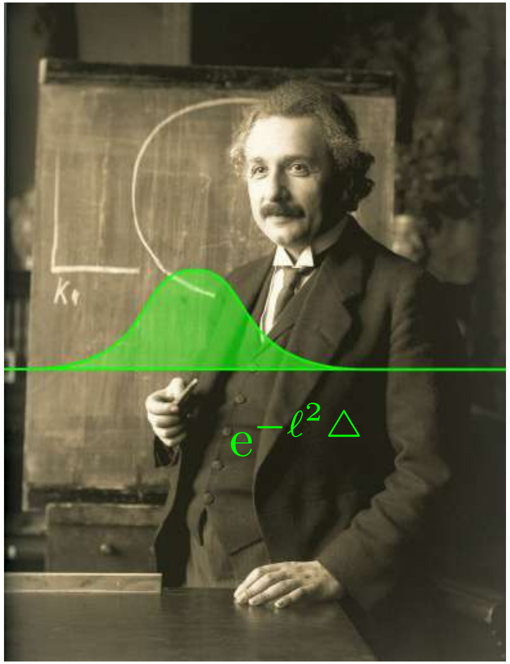

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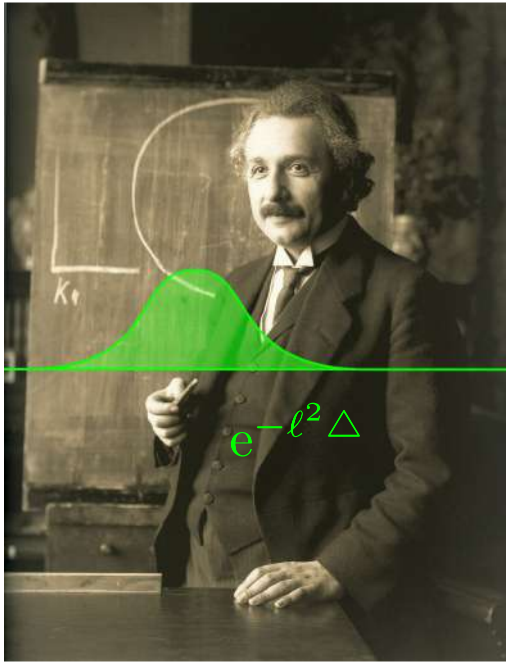

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Thank you to Luca and Sravan for their organization, to Valeri Frolov, Andrei Zelnikov, Jose Pinedo Soto, and Ivan Kolar for collaboration, and everybody for their attention.

Abstract

Regular solutions in weak-field infinite-derivative theories: Green function approach

In the weak-field regime it is sometimes possible to introduce notions of retarded non-local Green functions. A particular class of theories where this is possible has recently been dubbed "infinite-derivative theories." They typically feature non-local operators containing formal power series of Lorentz invariant differential operators. In this talk I will (i) introduce the notion of non-local causal Green functions and demonstrate that they satisfy DeWitt's "asymptotic causality" criterion, (ii) give an overview over regular weak-field solutions in infinite-derivative gravity, and (iii) comment on spacetime aspects involving the regularity of a uniformly accelerated source in a non-local theory. If time permits, I will highlight the problematic role of contour integration and Wick rotation in such non-local theories and comment on possible workarounds.

Based on work with Valeri Frolov, Andrei Zelnikov, Jose Pinedo Soto, Ivan Kolar.